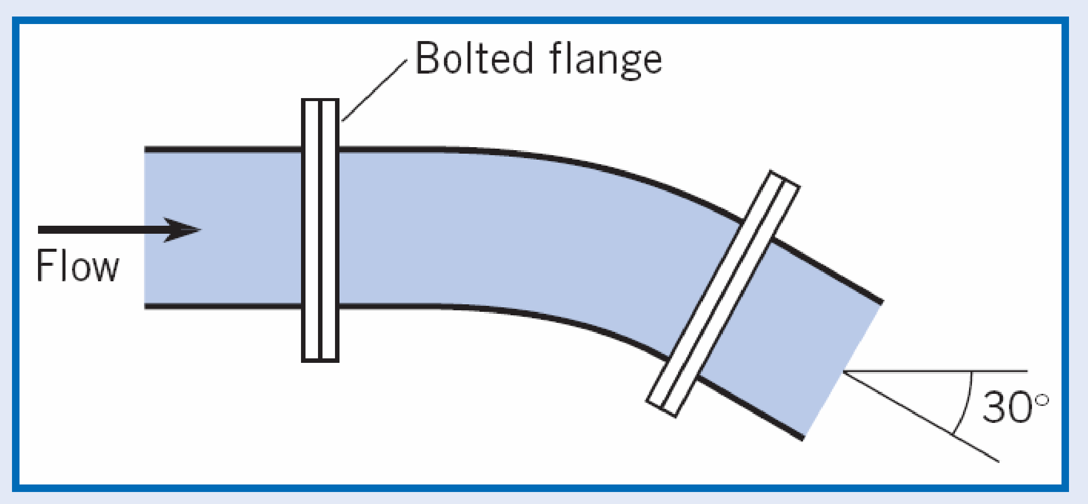
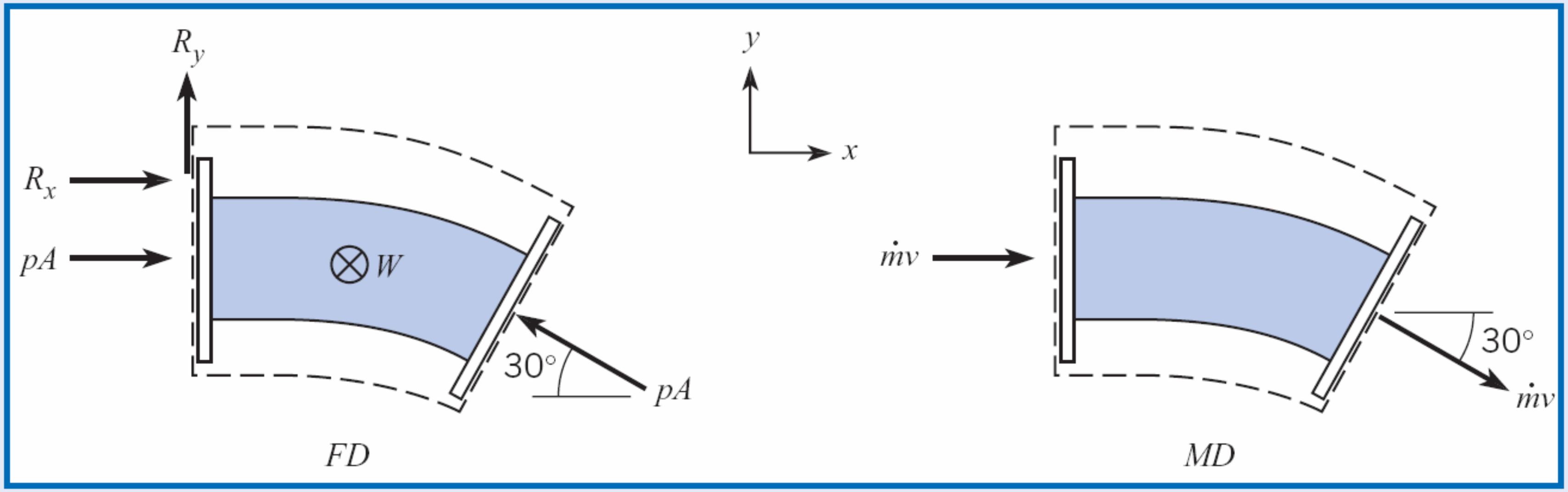
PIPES

Example 6.6

As shown in the figure, a 1-m-diameter pipe bend is carrying crude oil (S = 0.94) with a steady flow rate of 2 m^3/s. The bend has an angle of 30° and lies in a horizontal plane. The volume of oil in the bend is 1.2 m^3, and the empty weight of the bend is 4 kN. Assume the pressure along the centerline of the bend is constant with a value of 75 kPa gage. Find the net force required to hold the bend in place.



Find the force required to hold the bend in place?



This problem involves forces in the (x, y, z)

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$ (Since Flow is steady)

From the force diagram, $\sum F_x = R_x + pA - pA\cos 30$

$$\sum F_y = R_y + pA\sin 30$$

$$\sum F_z = R_z - W$$

From the momentum diagram, $V_{in} = V_{out}$ *From continuity* $(\dot{m} = \rho AV)$

$$\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in} = (\dot{m}v\cos 30) - (\dot{m}v)$$

$$R_x + pA - pA\cos 30 = \dot{m}v\cos 30 - \dot{m}v$$

$$\sum F_Y = \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY} = -(\dot{m}v\sin 60) - 0$$

$$R_y + pA\sin 30 = -\dot{m}v\sin 30$$

$$\sum F_Z = \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ} = 0$$

$$R_z - W = 0$$

Note: $W = \gamma Q + W_{bend}$

Resultant Force $\sum F = \sum F_x + \sum F_y + \sum F_z$

The net force $R = R_x + R_y + R_z$

The pressure force is

$$pA = (75\ \text{kN/m}^2)(\pi \times 0.5^2\ \text{m}^2) = 58.9\ \text{kN}$$

The fluid speed is

$$v = Q/A = \frac{(2\ \text{m}^3/\text{s})}{(\pi \times 0.5^2\ \text{m}^2)} = 2.55\ \text{m/s}$$

The momentum flow rate is

$$\dot{m}v = \rho Q v = (0.94 \times 1000\ \text{kg/m}^3)(2\ \text{m}^3/\text{s})(2.55\ \text{m/s}) = 4.80\ \text{kN}$$

The value of R_x is

$$R_x = -(pA + \dot{m}v)(1 - \cos 30°)$$
$$= -(58.9 + 4.80)(\text{kN})(1 - \cos 30°) = -8.53\ \text{kN}$$

The value of R_y is

$$R_y = -(pA + \dot{m}v)\sin 30°$$
$$= -(58.9 + 4.80)(\text{kN})(\sin 30°) = -31.8\ \text{kN}$$

The bend weight includes the oil plus the empty pipe:

$$W = \gamma V\!\!\!- + 4\ \text{kN}$$
$$= (0.94 \times 9.81\ \text{kN/m}^3)(1.2\ \text{m}^3) + 4\ \text{kN} = 15.1\ \text{kN}$$

So $R_z = 15.1$ kN. The net force acting on the bend to hold it stationary is

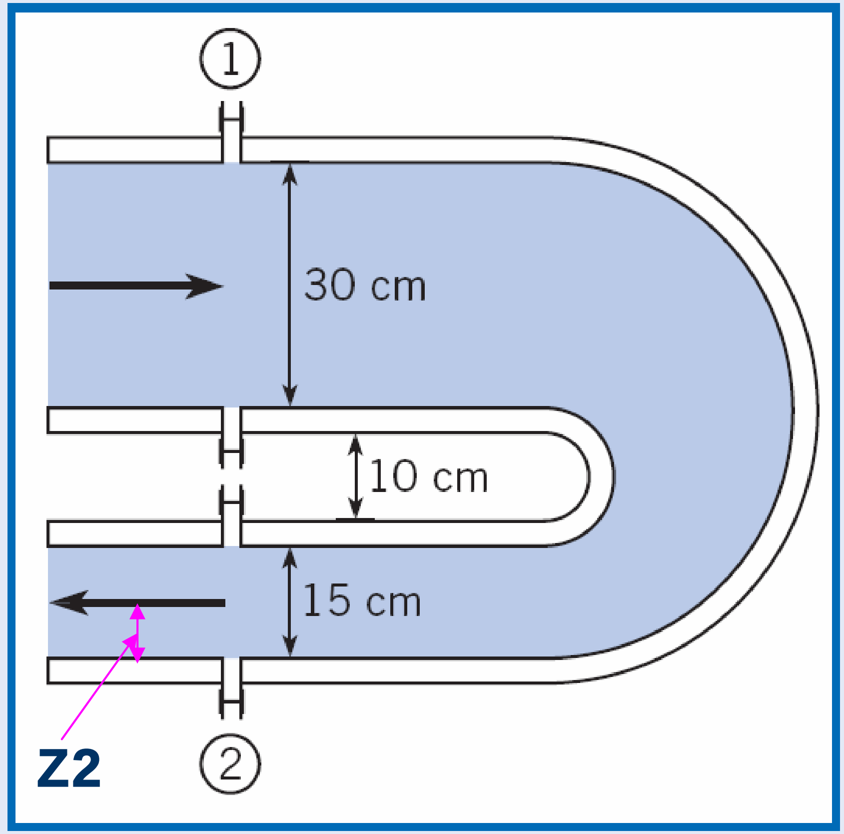
$$\mathbf{R} = (-8.53\ \text{kN})\mathbf{i} + (-31.8\ \text{kN})\mathbf{j} + (15.1\ \text{kN})\mathbf{k} \qquad \triangleleft$$

Example 6.7

Water flows through a 180° reducing bend, as shown. The discharge is 0.25 m^3/s, and the pressure at the center of the inlet section is 150 kPa gage. If the bend volume is 0.10 m^3, and it is assumed that the Bernoulli equation is valid, what force is required to hold the bend in place? The metal in the bend weighs 500 N.

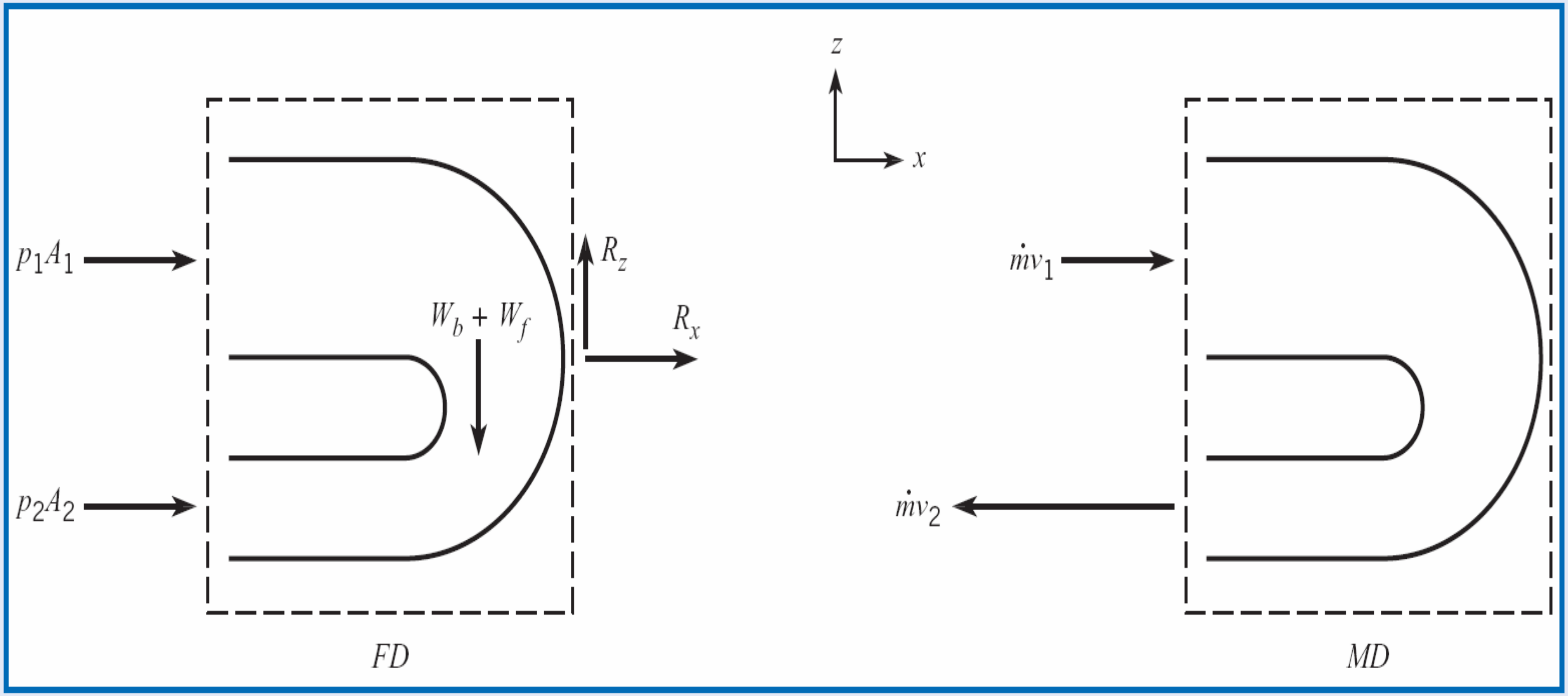
Find the force required to hold the bend in place?

Z1



Given:

$$\dot{Q}_2 = 0.25\, kg/m^3, \quad p_1 = 150\, kPa\ gauge, \quad Q_{bend} = 0.10\, m^3/s, \quad W_{bend} = 500\, N$$



This problem involves forces in the (x, z) directions

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_Z \rho dQ = 0$ (Flow is steady)

From the force diagram, $\sum F_x = R_x + p_1 A_1 + p_2 A_2$

$$\sum F_Z = R_z - W_b - W_f$$

From the momentum diagram, $\sum F_x = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in}) = -(\dot{m}v_2) - (\dot{m}v_1) = -\dot{m}(v_2 + v_1)$

$$\sum F_Z = \sum_{CS} (\dot{m}v)_{outZ} - \sum_{CS} (\dot{m}v)_{inZ} = o$$

$$R_x + p_1 A_1 + p_2 A_2 = -\dot{m}(v_2 + v_1)$$

$$R_z - W_b - W_f = 0$$

$$W_f = \gamma Q$$

Using Bernoulli's equation between section 1 & 2, we have,

$$p_1 + \gamma z_1 + \frac{1}{2}\rho v_1^2 = p_2 + \gamma z_2 + \frac{1}{2}\rho v_2^2$$

$$R_x + p_1 A_1 + p_2 A_2 = \dot{m}(v_2 - v_1)$$

$$R_z - W_b - W_f = 0 \qquad W_f = \gamma Q$$

From Continuity equation between section 1 & 2, we have,

$$Q = A_1 v_1 = A_2 v_2 = \frac{\pi d_1^2}{4} v_1^2 = \frac{\pi d_2^2}{4} v_2$$

(R_x, R_Z) **can be found**

Speeds are given by

$$v_1 = \frac{Q}{A_1} = \frac{0.25 \text{ m}^3/\text{s}}{\pi/4 \times 0.3^2 \text{ m}^2} = 3.54 \text{ m/s}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.25 \text{ m}^3/\text{s}}{\pi/4 \times 0.15^2 \text{ m}^2} = 14.15 \text{ m/s}$$

Mass flow rate is given by

$$\dot{m} = \rho Q = (1000 \text{ kg/m}^3)(0.25 \text{ m}^3) = 250 \text{ kg/s}$$

The net outward momentum flow rate is

$$\dot{m}(v_2 + v_1) = (250 \text{ kg/s})(14.15 + 3.54)(\text{m/s}) = 4420 \text{ N}$$

Pressure at section 2 is given by the Bernoulli equation:

$$p_2 = p_1 + \frac{\rho(v_1^2 - v_2^2)}{2} + \gamma(z_1 - z_2)$$

$$= 150\ \text{kPa} + \frac{(1000)(3.54^2 - 14.15^2)\text{Pa}}{2} + (9810)(0.325)\text{Pa}$$

$$= 59.3\ \text{kPa}$$

R_x is given by $R_x = -(p_1A_1 + p_2A_2) - \dot{m}(v_2 + v_1)$

The net pressure force is

$$p_1A_1 + p_2A_2 = (150\ \text{kPa})(\pi \times 0.3^2/4\ \text{m}^2) + (59.3\ \text{kPa})(\pi \times 0.15^2/4\ \text{m}^2)$$

$$= 11.6\ \text{kN}$$

The x component of the support force is

$$R_x = -(p_1A_1 + p_2A_2) - \dot{m}(v_2 + v_1)$$

$$= -(11.6\ \text{kN}) - (4.42\ \text{kN})$$

$$= -16.0\ \text{kN}$$

and the z component is

$$R_z = W_b + W_f$$

$$= 500\ \text{N} + (9810\ \text{N/m}^3)(0.1\ \text{m}^3)$$

$$= 1.48\ \text{kN}$$

END OF LECTURE (4)